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Yutao Chen

University of Padova

Shaoyuan Li

Shanghai Jiao Tong University

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Performance Analysis on Dynamic Matrix Controller with Single Prediction Strategy

YuTao Chen, ShaoYuan Li*, Ning Li
Department of Automation, Shanghai Jiao Tong University
Key Laboratory of System Control and Information Processing,
Ministry of Education, Shanghai, 200240, P.R.China
syli@sjtu.edu.cn

***Abstract*— The property of single prediction predictive control in the form of dynamic matrix control is studied within internal model control framework. The sensitivity function and integral squared error are used as performance evaluation criteria in the frequency and time domain respectively, to quantitatively analyze single prediction strategy, especially on controller with the prediction and control horizon $P = M = 1$. We present the correlation between system performance and model mismatch in this case. The performance limitation for tracking unit step signal is obtained through derivation and simulation.**

***Index Terms*— Dynamic Matrix Control, Internal Model Control, Sensitivity Function, Single Prediction Strategy, Performance Limitation**

I. Introduction

Model Predictive Control(MPC) is widely applied in industrial process control system for its inherent stability and excellent control performance. The main idea of MPC is to repeatedly solve a optimization problem online for a finite prediction horizon and then implement only the first control move at a certain time. While for engineers MPC is used to improve system performance and then profits, three problems with respect to MPC come into being in academia: the performance limitation(maximum achievable performance), the relationship between system performance and those design parameters and the advantages of MPC over other conventional control techniques. The attempts to solving these problems dated back to 1980s. Marchetti[1] and Maurath[3] presented a series of guidelines to design parameters for single-input single-output(SISO) systems based on simulation results. Marchetti also compared the performance between predictive controllers and PID controllers. These results are useful for controller design but contain few quantitative analysis.

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From the late 1980s, predictive control is also examined in the framework of Internal Model Control(IMC)[4]. IMC has its own properties which simplify the analysis on system performance. Morari[5] presents three main factors on system performance for SISO case: non-minimum phase(NMP) characteristics, control move constraints and model uncertainty. Sensitivity function and complementary sensitivity function are used as performance evaluation criteria. It is proved that "perfect control" is achieved for minimum phase system when the controller is selected to be the inverse of the model. DMC is one of techniques of MPC based on system step response modelling and its IMC form is discussed in detail by Xi[6] and Shu[7]. Therefore an analysis on performance limitation of DMC can be done in such a framework.

The goal of this paper is to approach quantitative analysis on performance of DMC based on IMC structure. For simplicity, a special strategy called Single Prediction Predictive Control(SPC) is studied. SPC is indicated by Marchetti[1] and Yuan[2] to has similar response from standard(multi-step prediction) strategies. The correlation between design parameters and system performance is provided. The limitation of the tracking error for a unit step signal is given through derivation and simulation

II. Performance evaluation criteria

Conventional closed-loop control system can be transformed into IMC structure equivalently if $q = c/(1 + p_m c)$, as shown in Fig.1, where d is disturbance signal, y, y_m plant and model outputs, c, q controller and its IMC controller, p, p_m plant and model transfer functions. Generally, system performance is evaluated both in frequency domain and time domain. We will go through these two parts respectively.

A. Frequency domain criterion

For nominal performance, sensitivity function measures the system's ability of reference signal tracking and disturbances restriction and is defined as

$$\epsilon \equiv \frac{e}{d-r} = \frac{y}{d} = \frac{1 - p_m q}{1 + q(p - p_m)} \quad (1)$$

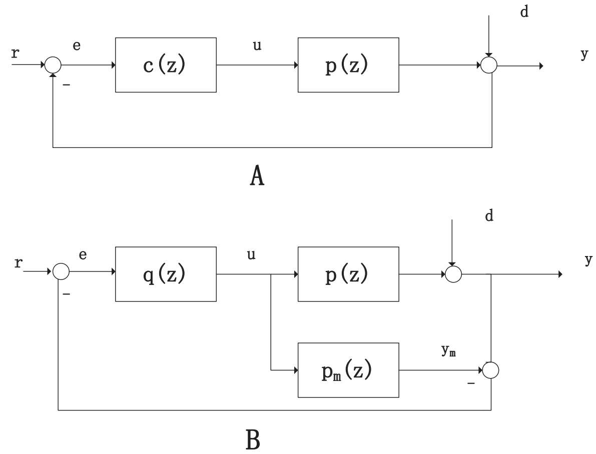

Fig. 1. General and IMC Block Diagram of Feedback Control System

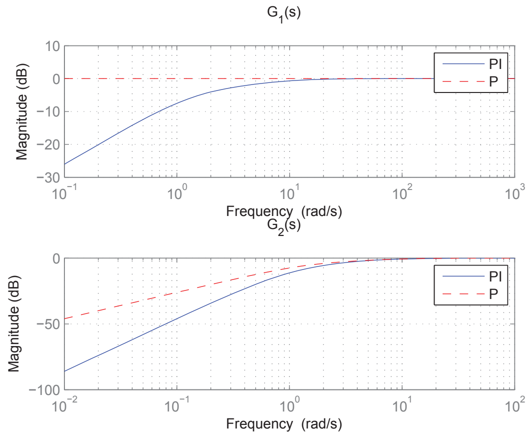

Fig. 2. Sensitivity Function with PI & P Controllers

$|\epsilon|$ is expected to be as small as possible and the control system achieves "perfect control" when $|\epsilon| = 0$. Morari indicates, however, that $|\epsilon|$ cannot equal to zero due to physical restrictions and its value varies as signal frequency increases. Therefore, it is reasonable to declare that a controller with smaller $|\epsilon|$ has a better performance. We will examine this by a simple example.

Consider two controllers for two plants: PI controller and P controller with an identical controller gain and two plants with $G_1(s) = \frac{1}{0.2s+1}$ and $G_2(s) = \frac{1}{0.2s^2+s}$. We expect that a PI controller is better than a P controller.

$$|\epsilon_P| = |\frac{1}{1 + p_m K_p}| \quad (2)$$

$$|\epsilon_{PI}| = |\frac{1}{1 + p_m K_p + \frac{K_p}{T_i} p_m \frac{1}{1-z^{-1}}}| \quad (3)$$

$$|\epsilon_P| > |\epsilon_{PI}| \quad (4)$$

The inequality (4) is satisfied in both(most) cases, as shown in Fig.2

A more simple but explicit criterion based on the sensitivity function is system bandwidth ω_B which is defined as

$$|\epsilon(\omega)| < 1/\sqrt{2} \qquad \forall \omega < \omega_B \quad (5)$$

Intuitively, a better system performance is expected with bigger system bandwidth.

For robustness, the complementary sensitivity function relates the reference r to the output y and is defined as

$$\eta \equiv \frac{y}{r} = \frac{pc}{1+pc} = 1 - \epsilon \quad (6)$$

According to the definition (6), η should be as close to unit as possible. However, model uncertainty imposes an upper bound on the magnitude of η [5]. As a result, η should be considered where model uncertainty exists.

By now, the sensitivity and complementary sensitivity function are two main factors in frequency domain when evaluating system performance.

B. Time domain criterion

It is also desirable to examine system performance under time domain. In this paper, H_2-optimal control formulation is adopted and the system performance is decided by the output tracking error

$$||e||_2^2 = \sum_{k=0}^{\infty} e_k^2 \quad (7)$$

where $e = y - r = \epsilon \times (d - r)$. Apart from sign, d and r have the same effect on e. Thus we denote the external input as $v = d - r$. According to Parseval Theorem, the error can be rewritten as

$$||e||_2^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\epsilon(e^{i\theta}) v(e^{i\theta}))|^2 d\theta \quad (8)$$

Note that there is no item of control move constraint in this criterion.

III. Performance Evaluation for DMC Based on Single Prediction Strategy

DMC in IMC structure is shown in Fig.3. In general, all design parameters have effects on system performance and stability including prediction horizon P, control horizon M, weighting items Q and R. In this paper, only one control move($M = 1$) at time k is calculated with one predictive output at a single time point $k + P$. The obtained control move is supposed to be invariant within the following P time intervals. Such strategy is called single prediction predictive control(SPC). In the following, we first give fundamental

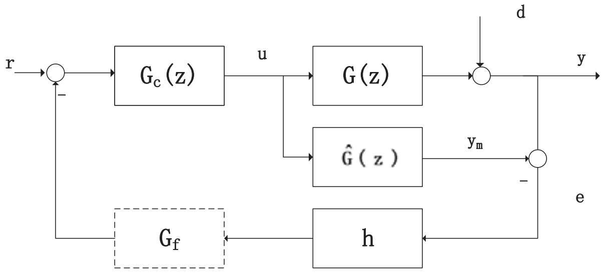

Fig. 3. DMC Block Diagram in IMC structure

descriptions for the model and controller. Then the frequency domain properties, ϵ and η are studied when model uncertainty exists. Finally the time domain property is derived. Illustrative examples support our results.

A. Model and control descriptions

For SPC, the model output is

$$y_m(k+P) = \sum_{i=1}^{N} \hat{g}_i u(k+P-i) \tag{9}$$
$$= \sum_{i=1}^{N} \hat{a}_i \triangle u(k+P-i) \tag{10}$$

where $g_i, a_i, i = 1, 2, \cdots, N$ are model impulse and step response respectively and

$$\hat{g}(z^{-1}) = \sum_{i=1}^{N} \hat{g}_i z^{-i+1} \tag{11}$$
$$\hat{a}(z^{-1}) = \sum_{i=1}^{N} \hat{a}_i z^{-i+1} \tag{12}$$

The transfer function of the plant and model will be

$$G(z^{-1}) = z^{-1} g(z^{-1}) \tag{13}$$
$$\hat{G}(z^{-1}) = z^{-1} \hat{g}(z^{-1}) \tag{14}$$

where $g = \sum_{i=1}^{N} g_i z^{-i+1}$ is plant impulse response. The predictive output and the corresponding function to minimize is

$$y_p(k+P) = y_m(k+P) + h[y(k) - y_m(k)] \tag{15}$$
$$J_p = q[y_p(k+P) - r(k+P)]^2 + \lambda \triangle u^2(k) \tag{16}$$

Function (16) can be rewritten as following if $u(k) = u(k+1) = \cdots = u(k+P)$

$$J_p = q\{\hat{G}_p[u(k-1) + \triangle u(k)] + \sum_{i=1}^{N-P} \hat{g}_{P+i} u(k-i) + he(k) - r(k+P)\}^2 + \lambda \triangle u^2(k) \tag{17}$$

where

$$\hat{G}_p = \sum_{i=1}^{P} \hat{g}_i = \hat{a}_p \tag{18}$$

Let $\partial J_p / \partial \triangle u(k) = 0$ with $\triangle u(k) = u(k) - u(k-1)$, we obtain the optimal control law

$$u(k) = \frac{1}{F(z^{-1})}[r(k+P) - e(k)] \tag{19}$$
$$F(z^{-1}) = G_p + \sum_{i=1}^{N-P} \hat{g}_{P+i} z^{-i} + (1 - z^{-1}) \frac{\lambda}{q G_p} \tag{20}$$

The controller is

$$G_c(z^{-1}) = \frac{1}{F(z^{-1})} \tag{21}$$

With (13)(14)(21), we are able to investigate the properties of SPC within IMC framework. In general, a filter G_f is set in feedback loop to tune system's output, stability and robustness. We simply set it to unit here to emphasize the effect of the controller G_c. Our results are given by several theorems.

Theorem 1: The system output is zero-offset for tracking step reference signal.

Proof: Suppose the reference input is $v = \frac{k}{1-z^{-1}}$, where k is an arbitrary step gain. For steady state conditions we have

$$F(1) = \hat{g}(1) \tag{22}$$
$$\epsilon(1) = \frac{1 - \hat{g}(1) F^{-1}(1)}{1 + F^{-1}(1)(g(1) - \hat{g}(1))} = 0 \tag{23}$$
$$e(\infty) = \lim_{z \to 1} (1 - z^{-1})(\epsilon * v) = 0 \tag{24}$$

■

Theorem 1 proves the zero-offset property of SPC even with model mismatch and control move weighting. Notice that equation (24) is not satisfied for all references. The controller here is designed using step response model and the input v has no zeros only when dealing with step reference. SPC cannot track ramp signal without offset.

B. One-time-interval prediction($P = 1$)

In this case, we predict the output at time $k+1$. The control law and the controller transfer function is

$$\triangle u^*(k) = u(k) - z^{-1} u(k) \tag{25}$$
$$= \frac{1}{\hat{a}(z^{-1}) + \frac{\lambda}{q \hat{a}_1}}[r(k+1) - e(k)] \tag{26}$$

$$G_c(z^{-1}) = \frac{1}{1 - z^{-1}} \cdot \frac{1}{\hat{a}(z^{-1}) + \frac{\lambda}{q \hat{a}_1}} \tag{27}$$
$$= \frac{1}{\hat{g}(z^{-1}) + (1 - z^{-1}) \frac{\lambda}{q \hat{a}_1}} \tag{28}$$

Theorem 2: If $\lambda = 0$, one-time-interval prediction controller is the inverse of model transfer function with one time interval delay. System performance is decided by the degree of model mismatch.

Proof: When $\lambda = 0$, (28) becomes

$$G_c(z^{-1}) = \frac{1}{\hat{g}(z^{-1})} = z^{-1}\hat{G}^{-1}(z^{-1}) \quad (29)$$

$$\epsilon_{DMC} = \frac{(1-z^{-1})\hat{g}}{\hat{g} + z^{-1}(g-\hat{g})} \quad (30)$$

Since g and $\hat{g}$ remain unchanged once the plant and model are selected, the sensitivity function ϵ is uniquely decided by their difference $g - \hat{g}$. Suppose there exists only gain mismatch $g = \mu\hat{g}$, where μ is the parameter of model mismatch. We have

$$\epsilon_{DMC} = \frac{1-z^{-1}}{1+(\mu-1)z^{-1}} \quad \mu > 0 \quad (31)$$

■

There are three different cases.

1) $\mu < 1$: The magnitude of sensitivity function increases when μ becomes smaller but the system remains stable.
2) $\mu > 1$: The magnitude of sensitivity function increases when μ becomes bigger. Mathematically, $\epsilon \rightarrow 0$ when $\mu \rightarrow \infty$. However, Xi[6] gives a upper bound of μ to ensure the stability of the system.
3) $\mu = 1$: The model matches the plant perfectly. The sensitivity function $\epsilon_{DMC} = 1 - z^{-1}$. Note that the sensitivity function has nothing to do with the form of plant and model.

It should be noted that one-time-interval prediction strategy cannot be implemented into NMP plants, i.e. NMP zeros and time delay. The reason is that the controller will be unstable if the plant has NMP zeros and for time delay system, $a_1 = 0$ makes controller G_c no sense.

Xi[6] demonstrates that for $G_f = 1$, $0 < \mu < 2$ is sufficient to ensure the closed-loop stability. Thus when $\mu \rightarrow 2$ the sensitivity function ϵ reaches its limit.

$$|\epsilon| \rightarrow |\frac{1-z^{-1}}{1+z^{-1}}| < |1-z^{-1}| = |\epsilon|_{\mu=1} \quad (32)$$

It is interesting to note that the sensitivity function when there is model mismatch is smaller than it when no model uncertainty exists, as shown in Fig.4. However, a further study on complementary sensitivity function has presented a more objective result on this problem. The sensitivity function ϵ cannot be the only criterion of performance estimation when model uncertainty exits. The complementary sensitivity function has to be added in this case. Fig.5 shows the complementary sensitivity function for above three cases. Poor robustness appears when $\mu > 1$. Now we can conclude that there is a tradeoff between system performance and robustness. Model mismatch, if exists, deteriorates system performance. Moreover, a too "weak" model gain may lead to instability.

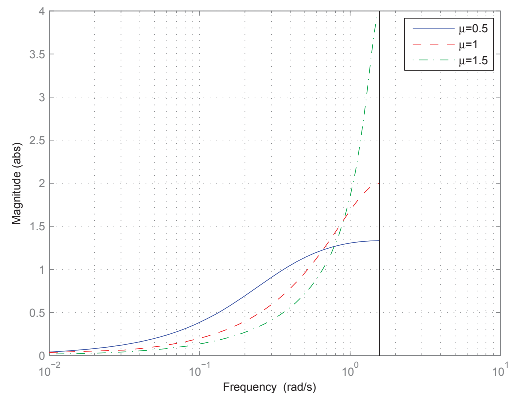

Fig. 4. Sensitivity Function $|\epsilon|$ with $\mu = 0.5, 1$ and 1.5

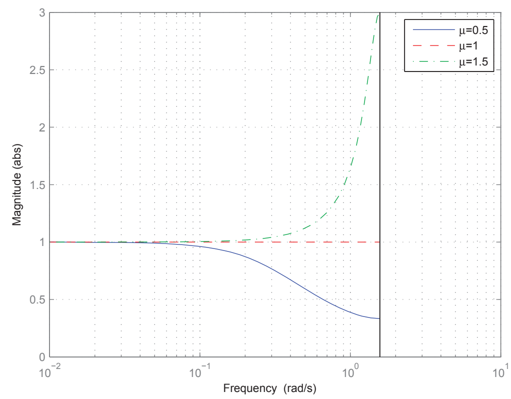

Fig. 5. Complementary Sensitivity Function $|\eta|$ with $\mu = 0.5, 1$ and 1.5

So far, SPC properties under frequency domain is studied. Next we move to time domain properties, mainly about calculating the tracking error $||e||_2^2$.

Theorem 3: If $\lambda = 0$ and $g = \hat{g}(\mu = 1)$, $||e||_2^2 = 1$

Proof: Substitute (31) into (8) with unit step input $v = \frac{z}{z-1}$, then

$$||e||_2^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |\frac{e^{i\theta}}{e^{i\theta}+(\mu-1)}|^2 d\theta \quad (33)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\frac{e^{i\theta}}{e^{i\theta}}|^2 d\theta = 1 \quad (34)$$

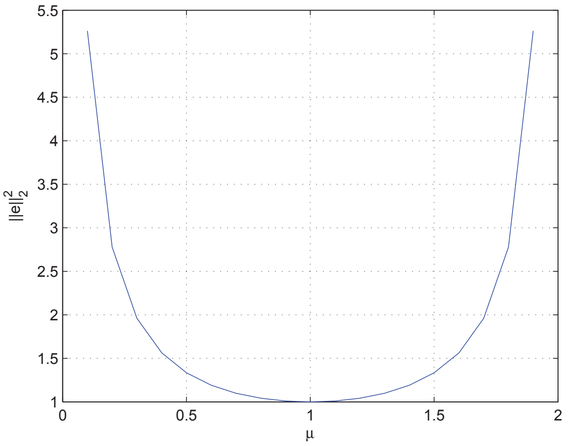

Fig. 6. Integral Squared Error $||e||_2^2$ with $\mu \in (0, 2)$

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A general relationship between μ and $||e||_2^2$ is shown in Fig.6. It is seen that model gain mismatch from -50% to 50% cause small errors and the error increase dramatically when the gain mismatch is greater than 50%. Immediately, we obtain the minimum tracking error for one-time-interval prediction strategy. This limitation is identical to the case where a basic sampled-data controller is used. It means SPC is able to track the reference signal within one step.

As we have indicated above, the tracking error is also affected by the inputs, namely the type of reference signal. The following table lists the minimum tracking error for 3 types of input.

TABLE I
MINIMUM TRACKING ERROR FOR DIFFERENT INPUTS

input v	error$\|\|e\|\|_2^2$
z^{-l}	2
$\frac{k}{1-z^{-1}}$	k^2
$\frac{z^{-1}}{(1-z^{-1})^2}$	Inf

In this section, some quantitative results are obtained for one-time-interval prediction case. Limitation of tracking unit step signal is calculated. However, given that a_1 is pretty close to zero, the controller G_c, even for minimum phase plants, is seldom applied in real systems.

C. Multi-time-interval prediction($P > 1$)

According to (20), the controller G_c has a item $\hat{a}_p$ instead of $\hat{a}_1$. As a result, this strategy is applicable in most cases. Now we examine the sensitivity function for the ideal case where $g = \hat{g}$ and $\lambda = 0$.

$$\epsilon = 1 - \frac{z^{-1}\hat{g}}{\hat{a}_p + \sum_{i=1}^{N-P} \hat{g}_{P+i} z^{-i}} \quad (35)$$

$$= \frac{\hat{a}_p + (1 - z^{-P}) \sum_{i=1}^{N-P} \hat{g}_{P+i} z^{-i}}{\hat{a}_p + \sum_{i=1}^{N-P} \hat{g}_{P+i} z^{-i}} \quad (36)$$

It is difficult to obtain an explicit formula that reflects the relationship between time interval P and system sensitivity function ϵ. A result is given for an extreme case where $P = N$.

$$G_c = \hat{a}_N^{-1} \quad (37)$$

The controller has become a P controller here. Hence, a strategy of N-time-interval prediction is suitable only for systems with good open-loop properties. Luo[10] presents robust analysis on this problem and demonstrates that N-time-interval prediction has the best robustness property among other time-interval strategies. When the prediction length P is between 1 and N, there is a trade-off between system robustness and performance. Fig.7 shows a system bandwidth test result for a system with monotonous step response, such as $G(s) = \frac{1}{0.5s+1}$. System bandwidth also expresses a monotonous property with prediction horizon. It is interesting to notice that ω_B approximately has the shape of $\frac{1}{x}$ with respect to P. ω_B is sensitive to horizon changes when P is small. Despite the intuitive and simulative results given above, theoretical demonstrations is still needed to explain the relationship between system performance and prediction horizon.

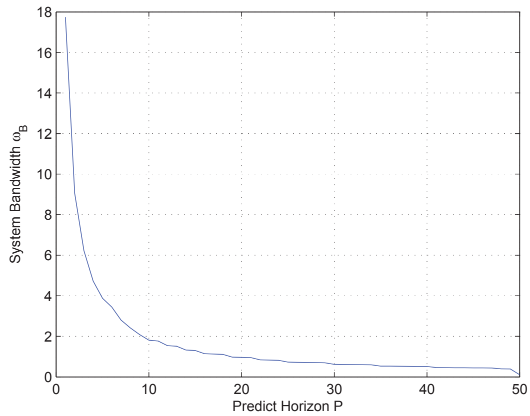

Fig. 7. Relations between Prediction Horizon P and System Bandwidth ω_B. The System is $G(s) = \frac{1}{0.5s+1}$

IV. CASE STUDY

In this section, the results obtained are tested by simulating a typical industrial process, which is used by Xi[6]. The system has the form of

$$G(s) = \frac{e^{-\tau s}}{1+s} \tag{38}$$

where $\tau = lT$. $l = 5$ is system time delay constant and T the sampling time period. The model transfer function is expressed as

$$G(z) = z^{-(l+1)} \frac{1-\sigma}{1-\sigma z^{-1}} \tag{39}$$

where $\sigma = e^{-T}$, and a SPC controller with prediction horizon P is given as (20) and (21).

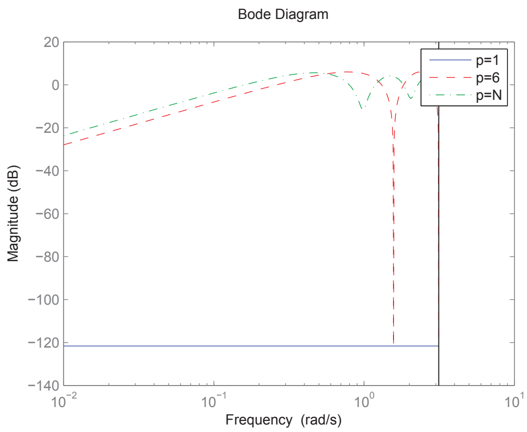

Fig. 8. The Sensitivity Function with $P = 1, 6$ and N

As shown in Fig.8, controller with prediction horizon less than system delay constant is unable to achieve desirable performance and system performance deteriorates as P increases. On the other hand, obvious spines are observed when P is small which means poor robustness. This is also supported by an investigation on complementary sensitivity function. The basic trade-off between nominal performance and robustness for selecting prediction horizon P is well illustrated in this case.

The integral squared error is computed and listed in TABLE II. As expected, the minimum ISE is achieved when $P = 6$, which represents an identical case where $P = 1$ for system without time delay. Theoretically, the value of minimum ISE is obtained as following. For time delay systems, the minimum tracking error is l[5]. This error together with the minimum ISE for SPC(see Theorem 3) form the final minimum ISE for this SPC controlled time delay system. Not surprisingly, the theoretical result coincides with the computed one($ISE = 6$).

TABLE II
MINIMUM TRACKING ERROR FOR DIFFERENT PREDICTION HORIZON

Prediction Horizon P	error$\|\|e\|\|_2^2$
6	6
8	6.154
10	6.170

V. CONCLUSION

The performance evaluation and its limitation for MPC have been matter of investigations for many years. While general results are quite difficult to obtain, a quantitative analysis on a special SPC strategy based on DMC is presented. The main idea is to use sensitivity function as a performance evaluation criterion to check the relationship between prediction horizon and system performance. For system with monotonous step response, prediction horizon is of inverse dependent to system bandwidth. The result is tested by a first order inertial system.

A special case of one-time-interval-prediction is studied in detail. The zero-offset property is demonstrated for tracking step reference signal in the presence of control variable soft constraint and model mismatch. We find that system performance is related but showing nonlinearities to the degree of model mismatch. A performance limitation is obtained in this case. The minimum tracking error to unit step signal is a constant 1, which coincide with the minimum error for a sampled-data control system.

These results have been tested by a typical industrial process with time delay. Results from theoretical derivations and MATLAB simulations is identical. It proves the correctness of our work. The methodology could be used for analysis on NMP and multi-input multi-output systems, where it is interesting to investigate how MPC affects system NMP zeros and their interactions with inputs.

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